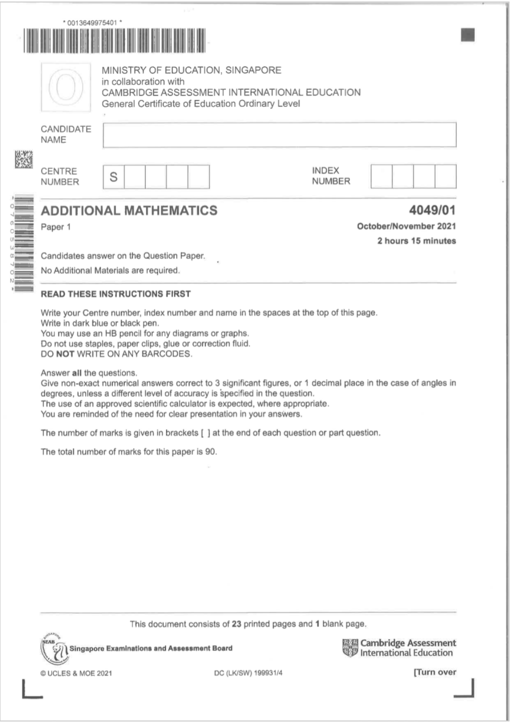
2021 O Level - A Maths P1 Solutions

Monday, 25 October 2021 5:30 PM

2021 4049 ADD MAT...



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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME	
CENTRE NUMBER	S
INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1

October/November 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **23** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

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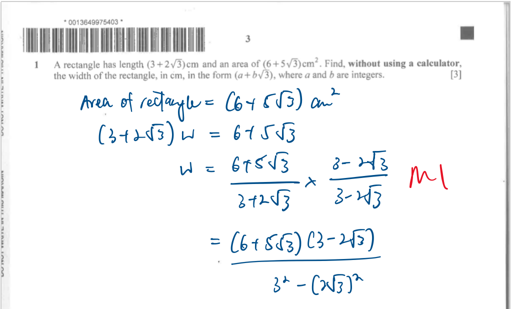

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1 A rectangle has length $(3+2\sqrt{3})$ cm and an area of $(6+5\sqrt{3})\text{cm}^2$. Find, **without using a calculator**, the width of the rectangle, in cm, in the form $(a+b\sqrt{3})$, where a and b are integers. [3]

Area of rectangle = $(6+5\sqrt{3})$ cm^2

$$(3+2\sqrt{3})\,w = 6+5\sqrt{3}$$

$$w = \frac{6+5\sqrt{3}}{3+2\sqrt{3}} \times \frac{3-2\sqrt{3}}{3-2\sqrt{3}} \qquad \text{M1}$$

$$= \frac{(6+5\sqrt{3})(3-2\sqrt{3})}{3^2-(2\sqrt{3})^2}$$

$$= \frac{18 - 12\sqrt{3} + 15\sqrt{3} - 10(3)}{9 - 12}$$ M1

$$= \frac{3\sqrt{3} - 12}{-3}$$

$$= 4 - \sqrt{3}$$

∴ width of rectangle $= (4 - \sqrt{3})$ cm A1

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2 The curve $xy + 10 = 0$ and the line $x + 2y + 1 = 0$ intersect at the points P and Q. Find the x-coordinate of P and of Q. [3]

$xy + 10 = 0$ — ①

$x = -2y - 1$ — ②

Sub ② into ①,

$(-2y - 1)y + 10 = 0$

$-2y^2 - y + 10 = 0$

$2y^2 + y - 10 = 0$

M1

$(y - 2)(2y + 5) = 0$

$y - 2 = 0$ or $2y + 5 = 0$

$y = 2$ $y = -\frac{5}{2}$

Sub $y = 2$ into ②,

$x = -2(2) - 1$

$x = -5$

Sub $y = -\frac{5}{2}$ into ②,

$x = -2(-\frac{5}{2}) - 1$

$x = 4$

∴ The x-coordinates of P and Q are $x = -5$ and $x = 4$.

A1 A1

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3 Express $3-12x-2x^2$ in the form $a(x+b)^2+c$ and hence state the coordinates of the turning point of the curve $y=3-12x-2x^2$. [4]

$$3-12x-2x^2 = -2\left(x^2+6x-\frac{3}{2}\right) \quad M1$$

$$= -2\left[(x+3)^2-\frac{21}{2}\right] \quad M1$$

$$= -2(x+3)^2+21 \quad M1$$

$\therefore$ Turning point of $y=3-12x-2x^2$ is $(-3, 21)$ A1

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4 Integrate $\frac{3}{x^2}+\frac{4}{3x-5}$ with respect to x. [4]

$$\int \frac{3}{x^2}+\frac{4}{3x-5}\,dx = \int 3x^{-2}\,dx + 4\int \frac{1}{3x-5}\,dx \quad M1$$

$$= -3x^{-1} + 4\left[\frac{1}{3}\ln(3x-5)\right] + C \quad M1$$

$$= -\frac{3}{x} + \frac{4}{3}\ln(3x-5) + C$$

A1 A1

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5 Express $\frac{13x-6}{x^2(2x-3)}$, in partial fractions. [6]

$$\frac{13x-6}{x^2(2x-3)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x-3}$$ M1

$13x-6 = Ax(2x-3)+B(2x-3)+Cx^2$ M1

Sub $x=0$,

$-6=-3B$

$B=2$ M1

Sub $x=\frac{3}{2}$,

$13(\frac{3}{2})-6 = C(\frac{3}{2})^2$

$\frac{27}{2} = \frac{9}{4}C$

$C=6$ M1

Sub $x=1$,

$13(1)-6 = A(2-3)+2(2-3)+6(1)$

$A=-3$ M1

$$\therefore \frac{13x-6}{x^2(2x-3)} = -\frac{3}{x}+\frac{2}{x^2}+\frac{6}{2x-3}$$ A1

6 A polynomial, P, is $2x^3 - x^2 - 13x + k$, where k is a constant.

(a) Find the value of k given that P leaves a remainder of 6 when divided by $x - 2$. [2]

$P(x) = 2x^3 - x^2 - 13x + k$

$$\left.\begin{aligned} P(2) &= 6 \\ 2(2)^3 - (2)^2 - 13(2) + k &= 6 \end{aligned}\right] \text{M1}$$

$16 - 4 - 26 + k = 6$

$k = 20$ A1

(b) In the case where $k = -6$, the quadratic expression $2x^2 + ax - 3$ is a factor of P. Find the value of the constant a. [4]

$P(x) = 2x^3 - x^2 - 13x - 6$

Try $(x-3)$,

$P(3) = 2(3)^3 - (3)^2 - 13(3) - 6$

$= 0$

$\therefore (x-3)$ is a factor of $P(x)$. M1

$2x^3 - x^2 - 13x - 6 = (x-3)(2x^2 + bx + 2)$

By comparing coefficient of x^2,

$-1 = -6 + b$

$b = 5$

$\therefore 2x^3 - x^2 - 13x - 6 = (x-3)(2x^2 + 5x + 2)$ M1

$= (x-3)(2x+1)(x+2)$

$= (2x^2 - 5x - 6)(x+2)$ M1

Since $2x^2 - 5x - 6$ is a factor

by comparison,

$a = -5$

A1

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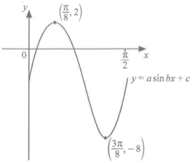

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The diagram shows the curve $y = a \sin bx + c$ for $0 \leqslant x \leqslant \frac{\pi}{2}$ radians. The curve has a maximum point at $\left(\frac{\pi}{8}, 2\right)$ and a minimum point at $\left(\frac{3\pi}{8}, -8\right)$.

(a) Explain why $c = -3$. [2]

Amplitude $= \frac{2-(-8)}{2}$

$= 5$

$-5 \leq 5 \sin bx \leq 5$

$-8 \leq 5 \sin bx - 3 \leq 2$ M1

$\therefore c = -3$ A1

(b) Explain why $b = 4$. [2]

Period $= \frac{\pi}{2}$

No. of cycles in 2π rad $= 2\pi \div \frac{\pi}{2}$ M1

$= 4$

$\therefore b = 4$ A1

(c) Hence find the equation of the curve. [2]

$y = 5 \sin 4x - 3$ A1

$\downarrow$

M1

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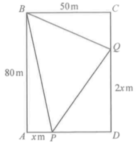

The diagram shows a rectangular field measuring 50 m by 80 m. Points P and Q lie on AD and CD respectively. Paths BP, PQ and QB cross the field. The triangular plot BPQ is to be planted with maize. It is given that $AP = x$ m and $DQ = 2x$ m.

(a) Show that the area, A m^2, to be planted with maize is given by

$$A = x^2 - 40x + 2000.$$

[3]

A = Area of □ ABCD − Area of △BCQ

− Area of △PQD − Area of △ABP M1

$= (50)(80) - \frac{1}{2}(50)\underline{(80-2x)} - \frac{1}{2}(2x)\underline{(50-x)} - \frac{1}{2}(80)(x)$ M1

$= 4000 - 2000 + 50x - 50x + x^2 - 40x$ M1

$= x^2 - 40x + 2000$ (shown)

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(b) Given that x can vary, find the stationary value of A and determine its nature. [4]

$$\frac{dA}{dx} = 2x - 40$$ M1

Let $\frac{dA}{dx} = 0$,

$2x - 40 = 0$

$2x = 40$

$x = 20$

M1

At $x = 20$,

$A = (20)^2 - 40(20) + 2000$

$= 1600$ A1

$\frac{d^2A}{dx^2} = 2 > 0$ (minimum).

∴ Stationary value of A is 1600

and it is minimum. A1

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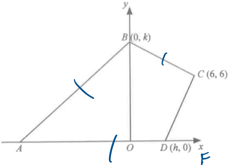

The diagram shows a kite $ABCD$ in which $AB = AD$ and $BC = CD$. Points A and D lie on the x-axis and B lies on the y-axis. Point C is (6, 6), D is $(h, 0)$ and B is $(0, k)$, where h and k are positive constants and $k \neq h$.

(a) Show that $h+k=12$. [3]

$$BC = CD$$

$$\sqrt{(k-6)^2+(0-6)^2} = \sqrt{(6-h)^2+(6-0)^2} \quad M1$$

$$(k-6)^2 = (6-h)^2$$

$k-6 = 6-h$ or $k-6 = -(6-h)$

$h+k = 12$ A1

(shown)

$k-6 = -6+h$

$k-h = 0$

$k = h$ A1

(rej)

It is now given that $h = 4$.

(b) Find the coordinates of A. [4]

$h+k=12$

$4+k=12$

$k=8$ M1

$m_{BD} = \frac{8-0}{0-4}$

$= -2$

$m_{AC} \times m_{BD} = -1$ M1

$m_{AC} = \frac{-1}{-2}$

$= \frac{1}{2}$

Let A be $(p, 0)$.

$m_{AC} = \frac{1}{2}$

$\frac{6-0}{6-p} = \frac{1}{2}$ M1

$12 = 6-p$

$p = -6$

$\therefore A(-6, 0)$ A1

(c) Find the area of the kite. [2]

$$\text{Area of kite} = \frac{1}{2}\left|\begin{matrix} 4 & 6 & 0 & -6 & 4 \\ 0 & 6 & 8 & 0 & 0 \end{matrix}\right| \quad M1$$

$$= \frac{1}{2}\{[4\times6 + 6\times8 + 0\times0 + (-6)(0)] - [0\times6 + 6\times0 + (8)(-6) + 0\times4]\}$$

$$= \frac{1}{2}[72-(-48)]$$

$= 60$ units^2 A1

10 (a) Prove the identity $\frac{\sin\theta}{1-\cos\theta}-\frac{1}{\sin\theta}=\cot\theta$. [4]

$$LHS = \frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta}$$

$$= \frac{\sin^2\theta - (1-\cos\theta)}{\sin\theta(1-\cos\theta)} \quad M1$$

$$= \frac{\underline{1-\cos^2}\,\theta - 1 + \cos\theta}{\quad} \quad M1$$

$$= \frac{\cos\theta(1-\cos\theta)}{\sin\theta(1-\cos\theta)} \quad M1$$

$$= \frac{\cos\theta}{\sin\theta} \quad M1$$

$$= \cot\theta$$

$= RHS$ (shown)

(b) Hence solve the equation $\frac{\sin 2\theta}{1-\cos 2\theta}-\frac{1}{\sin 2\theta}=-2$ for $0° \leqslant \theta \leqslant 180°$. [4]

$$\frac{\sin 2\theta}{1-\cos 2\theta} - \frac{1}{\sin 2\theta} = -2 \qquad 0° \leq 2\theta \leq 360°$$

$\cot 2\theta = -2$ M1

$\tan 2\theta = -\frac{1}{2}$ (sin/cos quad)

Basic $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$= 26.565°$ M1

$2\theta = 180 - 26.565,\ 360 - 26.565$

$\theta = 76.717, 166.717$

$\approx 76.7°, 166.7°$ (1 dp)

A1 A1

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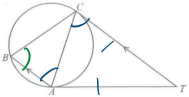
CT



In the diagram, A, B and C lie on a circle. The tangents to the circle at A and C meet at T and BA is parallel to CT.

(a) Prove that triangle ABC is isosceles. [4]

Let $\angle TCA = x°$

$\angle BAC = \angle TCA$ (alt. ∠s, $BA \parallel CT$) M1

$= x°$

$\angle ABC = \angle TCA$ (Alt. Segment Theorem) M1

$= x°$

$\therefore$ Since $\angle ABC = \angle BAC = x°$, M1

$\triangle ABC$ is isosceles (proven)

A1

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(b) Prove that angle BCA = angle CTA. [3]

$\angle BCA = 180 - \angle ABC - \angle BAC$ ($\angle$ sum of Δ)

M1

$= 180 - x - x$

$= (180 - 2x)^\circ$

$\angle CTA = 180 - \angle TCA - \angle TAC$ ($\angle$ sum of Δ)

M1

$= 180 - x - x$

$= (180 - 2x)^\circ$

$\therefore \angle BCA = \angle CTA = (180 - 2x)^\circ$ A1

(proven)

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12 (a) Show that the solution of the equation $6^x = 5 \times 3^{x+1}$ is $x = \frac{\lg 15}{\lg 2}$. [4]

$6^x = 5 \times 3^{x+1}$

$2^x \times 3^x = 5 \times 3^{x+1}$

$\frac{2^x}{5} = \frac{3^{x+1}}{3^x}$ M1

$\frac{2^x}{5} = 3^{x+1-x}$ M1

$\frac{2^x}{5} = 3$

$2^x = 15$

$\lg 2^x = \lg 15$

$x \lg 2 = \lg 15$ M1

$x = \frac{\lg 15}{\lg 2}$ (shown) A1

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(b) Express the equation $\log_3 x + \log_9 (x+2) = 2$ as a cubic equation in x. [4]

$\log_3 x + \log_9 (x+2) = 2$

$\log_3 x + \frac{\log_3 (x+2)}{\log_3 3^2} = 2$ M1

$\log_3 x + \frac{1}{2} \log_3 (x+2) = 2$

$\log_3 x + \log_3 (x+2)^{\frac{1}{2}} = 2$ M1

$\log_3 x\sqrt{x+2} = 2$

$3^2 = x(\sqrt{x+2})$ M1

$81 = x^2(x+2)$

$x^3 + 2x^2 - 81 = 0$ A1

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13 (a) A spherical balloon is being inflated with helium gas at a constant rate of 500 cm^3 per second. The balloon is initially empty.

(i) Find the radius of the balloon after one minute.
[The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.] [2]

let $V = \frac{4}{3}\pi r^3$

After 1 min,

$V = (500)(60)$ cm^3 M1

$\frac{4}{3}\pi r^3 = 30\,000$

$r = \sqrt[3]{\frac{22500}{\pi}}$

$= 19.2757$

≈ 19.3 cm (3 sf) A1

(ii) Find the rate at which the radius of the balloon is increasing after one minute. [3]

$\frac{dV}{dr} = 4\pi r^2$ M1

$\frac{dV}{dt} = 500$ cm^3/s

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

After 1 min,

$\frac{dr}{dt} = 500 \div [4\pi(19.2757)^2]$ M1

$= 0.107087$

≈ 0.107 cm/s (3 sf) A1

∴ Radius of balloon increases at 0.107 cm/s after 1 minute.

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(b) When a weather balloon is subject to different pressures, the pressure, P atmospheres, and the volume, V litres, of the air in the balloon are related by the formula $PV = k$, where k is a constant. Given that when $P = 1.2$, $V = 2$, find the rate at which V is changing with respect to P when $V = 2$. [4]

When $P = 1.2$, $V = 2$,

$k = (1.2)(2)$

$= 2.4$ M1

$V = \frac{2.4}{P}$ M1

$= \frac{12}{5}P^{-1}$

$\frac{dV}{dP} = -\frac{12}{5}P^{-2}$

$= -\frac{12}{5P^2}$ M1

When $V=2$, $P=1.2$,

$$\frac{dV}{dP} = -\frac{12}{5(1.2)^2}$$

$$= -1\tfrac{2}{3}$$ A1

∴ V changes w.r.t. P at a rate of $-1\tfrac{2}{3}$ litres per atmospheres.

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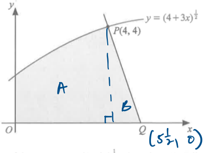

The diagram shows part of the curve $y=(4+3x)^{\frac{1}{2}}$. The point $P(4, 4)$ lies on the curve and the normal to the curve at P meets the x-axis at Q.

(a) Find the coordinates of Q. [5]

$y=(4+3x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(4+3x)^{-\frac{1}{2}}(3)$$

$$= \frac{3}{2\sqrt{4+3x}}$$ M1

At $P(4,4)$,

Gradient of tangent $= \dfrac{3}{2\sqrt{4+3(4)}}$

$$= \frac{3}{8}$$ M1

Gradient of normal $= -\frac{8}{3}$ M1

Eqn of normal:

$$y-4 = -\frac{8}{3}(x-4)$$

$$y = -\frac{8}{3}x + \frac{44}{3}$$ M1

Sub $y=0$,

$$0 = -\frac{8}{3}x + \frac{44}{3}$$

$$\frac{8}{3}x = \frac{44}{3}$$

$$x = 5\tfrac{1}{2}$$

∴ $Q(5\frac{1}{2}, 0)$ A1

(b) Find the area of the shaded region bounded by the curve, the normal PQ and the coordinate axes. [5]

Area of region A $= \int_0^4 (4+3x)^{\frac{1}{2}}\,dx$

$= \left[\dfrac{(4+3x)^{\frac{3}{2}}}{(\frac{3}{2})(3)}\right]_0^4$ M1

$= \left[\frac{2}{9}(4+3x)^{\frac{3}{2}}\right]_0^4$

$= \frac{2}{9}[4+3(4)]^{\frac{3}{2}} - \frac{2}{9}[4+3(0)]^{\frac{3}{2}}$ M1

$= \frac{112}{9}$ units^2

Area of region B $= \frac{1}{2}(1.5)(4)$ M1

$= 3$ units^2

$\therefore$ Total shaded region $= \frac{112}{9} + 3$ M1

$= 15\frac{4}{9}$ units^2 A1